Asymptotics and statistics on Fishburn matrices and their generalizations

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Fishburn matrices, introduced in the 1970s in the contexts of interval orders (in order theory) and directed graphs, are nonnegative, upper-triangular ones without zero row or zero column. They have later found to be bijectively equivalent to several other combinatorial structures such as ascent sequences, pattern-avoiding permutations, pattern-avoiding inversion sequences, Stoimenow matchings, and regular chord diagrams. In addition to their rich combinatorial connections, the corresponding asymptotic enumeration and the finer distributional properties are even more enriching and challenging, as will be presented in this talk. In particular, while the asymptotics of some classes of Fishburn matrices were known, the stochastic aspects of the major characteristic statistics have remained open up to now.

We develop a direct saddle-point analysis (without relying on any modular forms, identities or functional equations) to establish the asymptotics of Fishburn matrices and a large number of others with a similar sum-of-finite-product form for their (formal) general functions. In addition to solving several conjectures, the application of our saddle-point approach to the distributional aspects of statistics on Fishburn matrices is also examined with many new limit theorems characterized, representing the first of their kind for such structures.

This talk is based on joint work with Emma Yu Jin (University of Vienna).

Keywords: Generating function, asymptotic enumeration, saddle-point method, quantum modular forms, limiting distribution.

About Tiny-Pan Coin-Weighing Problem

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We study a special form of integer partition of n, where the target is to find a maximal partition P, with the maximum number of parts, that does not include any partition of f as its subset. Let $\rho(f, n)$ denote the number of parts in P. We show that such a problem is closely related to the tiny-pan coin weighing problem where each pan of the balance scale can hold exactly one coin, thereby deriving exact bounds for $\rho(f, n)$ in some of the cases. Furthermore, we show that $\rho(f, n)$ is ultimately periodic as a function of n.

This is a joint work with Te-Sheng Tan and Dai-Yang Wu.

Keywords: Coin Weighing Problem, Tiny Pan, Integer Partition

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Finding non-minority balls with majority and plurality queries

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Given a set of n colored balls, a *majority, non-minority or plurality ball* is one whose color class has size more than $n/2$, at least $n/2$ or larger than any other color class, respectively. We describe linear time algorithms for finding non-minority balls using query sets of size q of the following form: the answer to a majority/plurality query Q is a majority/plurality ball in Q or the statement that there is no such ball in Q.

This is a joint work with Dániel Gerbner and Balázs Patkós.

Keywords: search balls, sequential algorithm, majority queries.

The number of pattern occurrences in random planar maps

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The study of planar maps goes back to Tutte [12, 13] to mid-1960s. However, within the last 20 years there has been a renewed interest in the mathematical study of planar maps which started mainly due to the work by Schaeffer [10]. Recently there has also been a growing interest in local convergence [11] which means that a distribution of probability that a given local structure occurs around the root (or around a random vertex/ or anywhere in a random map) stabilizes.

The problem of pattern occurrences can be classified into two categories. One of them refers to local problems concerning pattern occurrences around the root [7, 9], the other involves global situations with patterns occurring anywhere in maps [2, 3, 5, 6]. Moreover, not only the pattern enumeration but also the submap enumeration is interested [1].

In this talk, I will present some results related to both local and global issues together with the history of planar maps. Whereas local problems can be completely solved, global problems is solved in general only in first order, which means that we can characterize the expected number of occurrences of a given pattern. In the global context, it is widely believed that the random variable that counts the number of occurrences of a pattern satisfies a central limit theorem [8].

Keywords: planar maps, pattern occurrence, generating functions, quadratic method, central limit theorem, local convergence.

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On the number of root ancestral configurations for a matching gene tree and species tree under uniform model and Yule model

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For a given pair of gene tree and species tree, the *ancestral configurations* for an internal node of the species tree are the set of distinct gene lineages that are present under the node. Results on the number of *root* ancestral configurations, which are the ancestral configurations for the root, under uniform model and Yule model will be discussed in this presentation. In particular, choosing both trees uniformly at random from the set of labeled topology with n leaves, the number of root ancestral configurations of such random trees is shown to asymptotically follow a lognormal distribution with mean $\sim 0.272n$ and variance $\sim 0.034n$. A similar result also holds for Yule model but with mean and variance of the number of root configurations grow asymptotically like $\sim 1.43^n$ and $\sim 2.04^n$. This is joint work with Filippo Disanto (University of Pisa), Michael Fuchs (National Chengchi University) and Noah Rosenberg (Stanford University).

Keywords: analytic combinatorics, gene trees, phylogenetics, species trees

Permutation statistics and signed statistics

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In this talk we give a brief survey on permutation statistics and present some recent works themed around signed statistics.

Keywords: permutation statistics, equidistribution, Euler numbers, Eulerian numbers, Coxeter groups, Mahonian, signed Mahonian, Euler-Mahonian

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Signed mahonian identities on permutations with subsequence restrictions

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In this talk, we will introduce some results surrounding Caselli's conjecture on the equidistribution of the major index with sign over the two subsets of permutations of $\{1, 2, \ldots, n\}$ containing respectively the word $12 \cdots k$ and the word $(nk + 1) \cdots n$ as a subsequence, under a parity condition of n and k. We derive broader bijective results on permutations containing varied subsequences. Hence, we obtain the signed mahonian identities on families of restricted permutations, in the spirit of a well-known formula of Gessel–Simion, covering a combinatorial proof of Caselli's conjecture. We also derive an extension of the insertion lemma of Han and Haglund–Loehr–Remmel which allows us to obtain a signed enumerator of the major-index increments resulting from the insertion of a pair of consecutive numbers in any place of a given permutation.

Keywords: signed major index, equidistribution, permutation with subsequence restrictions, linear extensions, pattern avoiding permutations, insertion lemma

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On the integer $\{k\}$-domination number of circulant graphs

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Let $G = (V, E)$ be a simple undirected graph. G is a circulant graph defined on $V = \mathbb{Z}_n$ with difference set $D \subseteq \{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor\}$ provided two vertices i and j in $\mathbb{Z}_n$ are adjacent if and only if $\min\{|i-j|, n-|i-j|\} \in D$. For convenience, we use $G(n; D)$ to denote such a circulant graph.

A function $f : V(G) \to \mathbb{N} \cup \{0\}$ is an integer $\{k\}$-domination function if for each $v \in V(G)$, $\sum_{u \in N_G[v]} f(u) \geq k$. By considering all $\{k\}$-domination functions f, the minimum value of $\sum_{v \in V(G)} f(v)$ is the $\{k\}$-domination number of G, denoted by $\gamma_k(G)$. In this paper, we prove that if $D = \{1, 2, \ldots, t\}$, $1 \leq t \leq \frac{n-1}{2}$, then the integer $\{k\}$-domination number of $G(n; D)$ is $\lceil \frac{kn}{2t+1} \rceil$. For more details please see [1]. This is a joint work with Yen-Jen Cheng and Hung-Lin Fu.

Keywords: Circulant graph, integer $\{k\}$-domination number, Euclidean algorithm, integer linear program.

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Multichannel Conflict-Avoiding Codes

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A conflict-avoiding code (CAC) $\mathcal{C}$ of length n and weight k is a collection of k-subsets of $\mathbb{Z}_n$ such that $\Delta(x) \cap \Delta(y) = \emptyset$ for any $x, y \in \mathcal{C}$ and $x \neq y$, where $\Delta(x) = \{a - b : a, b \in x, a \neq b\}$. CAC was introduced by Levenshtein and Tonchev [1] as a scheduling mechanism for multiple-access collision channel without feedback. Recently, we extend CACs to support multichannel wireless networks [2]. Upper bounds on the codewords for multichannel conflict-avoiding codes with weights three and four are derived, and optimal codes attaining these bounds are given.

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